

Thermoacoustic effect of traveling–standing wave

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ABSTRACT

In order to improve thermoacoustic efficiency, the thermoacoustic devices have been developed from standing wave devices to traveling wave devices. Actually, the acoustic field in practical thermoacoustic devices is neither a pure traveling wave nor a pure standing wave. The thermoacoustic effect is the hybrid effect of traveling wave component and standing wave component. Therefore, the thermoacoustic effect of traveling–standing wave will be study in this paper. Firstly, the thermoacoustic conversion performance of the traveling–standing wave are analyzes qualitatively by combining the thermoacoustic conversion performance of the traveling wave with those of the standing wave. Then, based on the basic thermoacoustic formulas, the influence of the parameters of the acoustic field and the regenerator's structure on the thermoacoustic conversion is analyzed, and the optimum condition for the thermoacoustic conversion is discussed. The results are consistent with the qualitative analysis. Additionally, our theoretical results also show a good agreement with the experimental data [Biwa et al. *Phys Rev E* 2004;69(6):066304(6)], which indicates the validity of the analysis in this paper. Furthermore, the analysis in this paper further provides a more intensive understanding of these experimental results. The conclusions obtained in this paper are significant to guide for the design of new thermoacoustic devices.

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1. Introduction

Thermoacoustic researches have made great progress over decades, and benefited greatly from the publication of thermoacoustic conversion theory. In the early time, various standing wave devices [1,2] were built based on thermoacoustic theory. In the standing wave device, gas parcels with $\varphi_z = (2n + 1) \times 90^\circ$ realize the conversion between heat and acoustic energy through irreversible thermal contact between the working gas and the stuff in the regenerator, where φ_z is the leading phase of oscillating pressure p_1 to oscillating velocity u_1 . However, the purposely imperfect thermal contact on which the standing wave thermoacoustic conversion relies limits its efficiency. With the development of thermoacoustic study, the traveling wave thermoacoustic device was proposed by Ceperley [3] in 1979. In such engine, gas parcels work at the leading phase $\varphi_z = 2n \times 90^\circ$ to realize the reversible thermoacoustic conversion in theory. However, the reversible thermoacoustic conversion realized by reversible thermal contact requires infinitesimal hydraulic radius of the regenerator. This will lead to infinite viscous dissipation which will weaken the thermoacoustic gain and efficiency strongly. So the finite hydraulic radius is adopted in the real traveling wave device, which makes the oscillating temperature of gas parcels lag the oscillating displacement of gas parcels. It indicates

that the phase difference between the oscillating temperature and the oscillating pressure deviates from $(2n + 1) \times 90^\circ$ because of the thermo-relaxation. Under such condition, for improving thermoacoustic gain and efficiency, the traveling–standing wave phase is necessary to match the phase deviation due to the thermo-relaxation. Biwa et al. [4] demonstrated thermoacoustic energy conversions by making full use of the acoustic field induced in the resonator. He claimed that, in order to achieve high gain and efficiency, one has to choose an optimum φ_z which can make both the traveling wave component and the standing wave component contribute to the amplification of acoustic intensity. However, the leading phase φ_z can be chosen in the region of $-180^\circ \leq \varphi_z \leq 180^\circ$ for the traveling–standing wave, then how to choose the optimum φ_z for the regenerator becomes a new problem. So far, most researchers still focus their particular attentions on the traveling wave [5,6] or the standing wave [7–10], and few published literatures have studied on the traveling–standing wave field.

Actually, the acoustic field in practical thermoacoustic devices is neither a pure traveling wave nor a pure standing wave. The thermoacoustic effect is the hybrid effect of the traveling wave component (TWC) and the standing wave component (SWC). The functions of the SWC are energy storage and energy conversion, while the functions of the TWC are energy transfer and energy conversion. In the thermoacoustic engine (TAE), the acoustic energy produced by the SWC can be transferred by the TWC, and the acoustic energy produced by the TWC also can be deposited with the SWC state. In the thermoacoustic cooler (TAC), the acoustic

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energy consumed by the SWC can be supplied by the TWC, and the acoustic energy consumed by the TWC can also be compensated from the stored acoustic energy in the SWC.

So, the thermoacoustic effect of traveling–standing wave will be discussed in this paper. The thermoacoustic conversion performance of the traveling–standing wave will be analyzed by combining the thermoacoustic conversion performance of the traveling wave with those of the standing wave. Then, based on the basic thermoacoustic formulas, the influence of the parameters of the acoustic field and the regenerator's structure on the thermoacoustic conversion is analyzed, and the optimum condition for the thermoacoustic conversion is discussed. Additionally, some numerical results comparing with the experimental data [4] will be provided, which will validate the analysis of this paper.

2. Qualitative analysis

The thermoacoustic effect is the hybrid effect of the TWC and SWC in the practical thermoacoustic system. So the conversion capabilities of the traveling wave and the standing wave will be analyzed firstly. And then, the thermoacoustic conversion performance of the traveling–standing wave by combining the thermoacoustic conversion performance of the TWC with those of the SWC will be discussed.

2.1. Standing wave mode

As shown in Fig. 1, the regenerator is located on the left side of the pressure antinode (PAN). Taking the rightward direction as the positive direction of x , the leading phase φ_z of the acoustic pressure p_1 relative to the velocity u_1 equals to -90° in the regenerator. In x direction, the heat power $\dot{Q}_2$ transferred by the regenerator can be expressed as [1]

$$\dot{Q}_2 = -\frac{1}{4}\Pi\delta_k T_m \beta p_1 u_1 (\Gamma - 1), \quad (1)$$

and the produced acoustic power $\dot{W}_2$ is expressed as [1]

$$\dot{W}_2 = \frac{1}{4}\Pi\delta_k \Delta x \frac{T_m \beta^2 \omega}{\rho_m c_p} (p_1)^2 (\Gamma - 1), \quad (2)$$

where Π is the perimeter, Δx is the length of the regenerator, ω is the angular frequency, β is the thermal expansion coefficient, δ_k and δ_v are the thermal penetration depth and the viscous penetration depth respectively, c_p is the isobaric specific heat, ρ_m is the mean density of the working gas, T_m is the cross-section averaged temperature of the fluid, and $\Gamma = \nabla T_m / T_{crit}$ is the ratio of actual temperature gradient ∇T_m to the standing wave critical gradient $T_{crit} = T_m \beta p_1 / \rho_m c_p u_1$.

As shown in Eqs. (1) and (2), the heat power and the produced acoustic power are proportional to the temperature gradient factor $(\Gamma - 1)$. When $\nabla T_m = \nabla T_{crit}$ (i.e. $(\Gamma - 1) = 0$), there is neither heat power transferred nor acoustic power produced. For $\nabla T_m > \nabla T_{crit}$ (i.e. $(\Gamma - 1) > 0$), the heat power is transferred away from the PAN, and the acoustic power is produced in the regenerator. For $\nabla T_m < \nabla T_{crit}$ (i.e. $(\Gamma - 1) < 0$), the heat power is transferred to-

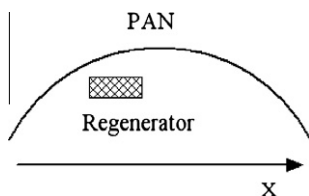


Fig. 1. In the standing wave, the regenerator is located on the left side of the PAN. The positive direction of x is taken the rightward direction.

ward the PAN, and the acoustic power is consumed in the regenerator. There are two cases for $\nabla T_m < \nabla T_{crit}$. In the case of $0 < \nabla T_m < \nabla T_{crit}$, the regenerator consumes acoustic power to pump heat from the cold end to the hot end. In the other case of $\nabla T_m \leq 0$, the regenerator dissipates acoustic power, and the heat power is transferred from the hot end to the cold end. In that case, the regenerator will not realize the conversion from heat to acoustic energy, and there is also no heat pumping from the cold end to the hot end. The regenerator located on the right side of the PAN can be analyzed in the same way.

Therefore, some conclusions can be obtained as shown in Fig. 2. The required working conditions for the standing wave engine are: (1) the hot end of the regenerator is close to the PAN; (2) $\nabla T_m > \nabla T_{crit}$ on the left side of the PAN, or $\nabla T_m < -\nabla T_{crit}$ on the right side of the PAN. The required working conditions for the standing wave cooler are: (1) the ambient end of the regenerator should be close to the PAN; (2) $0 < \nabla T_m < \nabla T_{crit}$ on the left side of the PAN, or $-\nabla T_{crit} < \nabla T_m < 0$ on the right side of the PAN.

2.2. Traveling wave mode

The traveling wave thermoacoustic device was originally proposed by Ceperley in 1979 [3]. In the traveling wave heat engine, the heat flow direction is opposite to the direction of wave propagation: the wave propagates from the cold end to the hot end, while the heat transfers from the hot end to the cold end. The regenerator converts heat to acoustic energy when the wave propagates from the cold end to the hot end. In a traveling wave heat pump, the wave pumps heat in the opposite direction to its propagation: the wave propagates from the hot end to the cold end, while the heat is pumped from the cold end to the hot end. The regenerator consumes the acoustic energy to pump heat from the cold end when the wave propagates from the hot end to the cold end. Thus, the regenerators of the TAE and the TAC in the traveling wave should be arranged according to Fig. 3.

2.3. Traveling–standing wave mode

We can analyze the thermoacoustic effect of the regenerator arranged as the four cases in Fig. 4 by combining the thermoacoustic conversion performance of traveling wave and standing wave.

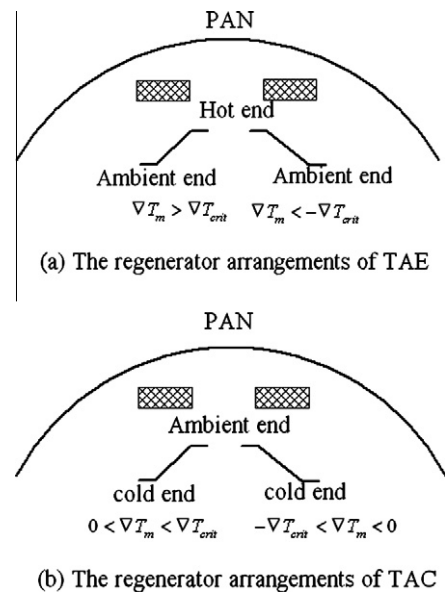


Fig. 2. In the standing wave, the better arrangements of regenerators.

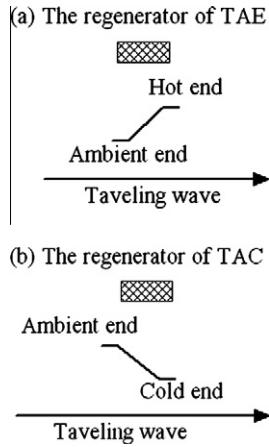


Fig. 3. In the traveling wave, the better arrangements of regenerators.

Case 1. The hot end is close to the PAN, and the TWC propagates from the cold end to the hot end.

For the TAE, when $\nabla T_m > \nabla T_{crit}$, the regenerator realizes the conversion from heat to acoustic energy by both the SWC and the TWC, which can improve the thermoacoustic gain and efficiency.

For the TAC, when $0 < \nabla T_m < \nabla T_{crit}$, the SWC pumps heat from the cold end to the hot end, while the TWC transfers heat from the hot end to the cold end. The direction of the heat transfer induced by SWC is opposite to that induced by TWC. When the thermoacoustic effect induced by the SWC is stronger than that by the TWC, the regenerator can pump heat from the cold end.

Case 2. The cold end is close to the PAN, and the TWC propagates from the cold end to the hot end.

For the TAE, the SWC consumes acoustic energy, while the TWC converts heat to acoustic energy. When the thermoacoustic effect induced by the TWC is stronger than that by the SWC, the regenerator can produce acoustic energy.

For the TAC, heats are transferred from the hot end to the cold end by both the SWC and the TWC. The regenerator cannot pump heat from the cold end.

Case 3. The hot end is close to the PAN, and the TWC propagates from the hot end to the cold end.

For the TAE, when $\nabla T_m > \nabla T_{crit}$, the SWC converts heat to acoustic energy, while the TWC consumes acoustic energy. When the thermoacoustic effect induced by the SWC is stronger than that by the TWC, the regenerator can produce acoustic energy.

For the TAC, when $0 < \nabla T_m < \nabla T_{crit}$, the regenerator can pump heat from the cold end by both the SWC and the TWC, which can improve the thermoacoustic gain and efficiency.

Case 4. The cold end is close to the PAN, and the TWC propagates from the hot end to the cold end.

For the TAE, both the SWC and the TWC consume acoustic energy. The regenerator cannot realize the conversion from heat to acoustic energy.

For the TAC, the SWC transports heat from the hot end to the cold end, while the TWC pumps heat from the cold end to the hot end. When thermoacoustic effect induced by the TWC is stronger than that by the SWC, the regenerator can pump heat from the cold end.

To sum up, the regenerator arranged as Case (4) just consumes acoustic power. The regenerators arranged as Cases (1), (2) and (3) can produce acoustic power under certain conditions. Especially, Case (1) is the best arrangement for the TAE's regenerator in the four cases, because the regenerator of TAE can realize the conversion from heat to acoustic energy by both the SWC and the TWC. The regenerator arranged as Case (2) cannot pump heat from the cold end, while the regenerators arranged as Cases (1), (3) and (4) can pump heat under certain conditions. Especially, Case (3) is the best arrangement for the TAC's regenerator of the four cases, because the regenerator can pump heat from the cold end by both the SWC and the TWC.

3. Traveling-standing wave

Consider wave propagating in the x direction in an ideal gas within a channel with constant cross-sectional area A . The hydraulic radius is conventionally defined as [11]

$$r_h = A/\Pi. \quad (3)$$

In the resonator tube, $\delta_v \ll r_h$, $\delta_k \ll r_h$. So, in a channel with large cross-section, the viscosity due to the interaction between the acoustic wave and the solid channel wall can be neglected. With the first order of acoustic approximation, the x direction component of the momentum equation is

$$-\rho_m \frac{\partial u_1}{\partial t} = \frac{\partial p_1}{\partial x}, \quad (4)$$

In the ideal acoustic channel, the acoustic field can be described by the superposition of a rightward traveling wave with a leftward traveling wave. The oscillating pressure can be expressed as

$$\begin{aligned} p_1 &= \tau p_+ + (1 - \tau) p_- = p_a [\tau e^{i(\omega t - kx)} + (1 - \tau) e^{i(\omega t + kx)}] \\ &= p_a e^{i\omega t} [\tau e^{-ikx} + (1 - \tau) e^{ikx}], \end{aligned} \quad (5)$$

where k is the wave vector, p_a is the pressure amplitude, τ is defined as the traveling wave ratio, and $0.0 \leq \tau \leq 1.0$. $\tau = 0.0$ means the pure leftward traveling wave, $\tau = 0.5$ means the pure standing wave, $\tau = 1.0$ means the pure rightward traveling wave, $0.0 < \tau < 0.5$ means the mixed acoustic field of the SWC with the ratio of 2τ and the leftward TWC with the ratio of $(1 - \tau) - \tau$, and $0.5 < \tau < 1.0$ means the mixed acoustic field of the SWC with the ra-

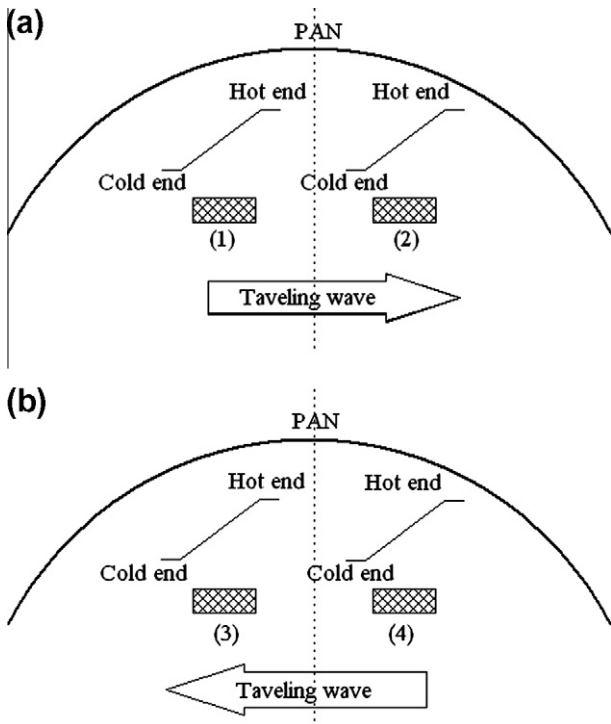


Fig. 4. The four cases of the regenerator arrangements.

tio of $2(1 - \tau)$ and the rightward TWC with the ratio of $\tau - (1 - \tau)$. Substituting Eq. (5) into Eq. (4)

$$u_1 = -\frac{1}{\rho_m} \int \frac{dp_1}{dx} dt = \frac{p_a e^{i\omega t}}{\rho_m c} [\tau e^{-ikx} - (1 - \tau)e^{ikx}], \quad (6)$$

where c is the sound speed.

According to Eqs. (5) and (6), the specific acoustic impedance can be yield,

$$Z = \frac{p_1}{u_1} = \rho_m c \times \frac{\tau e^{-ikx} + (1 - \tau)e^{ikx}}{\tau e^{-ikx} - (1 - \tau)e^{ikx}}. \quad (7)$$

The phase of Z represents the leading phase φ_z of p_1 to u_1 . Fig. 5 shows the results of the calculation for φ_z versus the normalized position kx/π for different τ . When $\tau = 0.0$, it is the pure leftward traveling wave, and $\varphi_z = \pm 180^\circ$. When $\tau = 0.5$, it is the pure standing wave, and $\varphi_z = -90^\circ$ in the region of $-\pi/2 < kx < 0$ and $\varphi_z = 90^\circ$ in the region of $0 < kx < \pi/2$. When $\tau = 1.0$, it is the pure rightward traveling wave, and $\varphi_z = 0^\circ$. When $0.0 < \tau < 0.5$, it is the mixed acoustic field of the SWC and the leftward TWC, and $-180^\circ < \varphi_z < -90^\circ$ in the region of $-\pi/2 < kx < 0$ and $90^\circ < \varphi_z < 180^\circ$ in the region of $0 < kx < \pi/2$. When $0.5 < \tau < 1.0$, it is the mixed acoustic field of the SWC and the rightward TWC, and $-90^\circ < \varphi_z < 0^\circ$ in the region of $-\pi/2 < kx < 0$ and $0^\circ < \varphi_z < 90^\circ$ in the region of $0 < kx < \pi/2$.

According to the analysis and Fig. 4, it is obtained that $-90^\circ < \varphi_z < 0^\circ$ for Case (1), $0^\circ < \varphi_z < 90^\circ$ for Case (2), $-180^\circ < \varphi_z < -90^\circ$ for Case (3), and $90^\circ < \varphi_z < 180^\circ$ for Case (4).

4. Thermoacoustic formula analysis

This section discusses the thermoacoustic effect of one point of the regenerator instead of the whole regenerator. Although, the acoustic field cannot keep invariant along the regenerator, the acoustic field in the regenerator can be regarded as a set of discrete points with variant acoustic characteristics. We will locate the regenerator in a certain region of acoustic field, which has better characteristics for thermoacoustic conversion.

4.1. Acoustic power

The average acoustic power $d\dot{E}_2$ produced in the length dx of a regenerator is expressed as [11],

$$\frac{d\dot{E}_2}{dx} = -\frac{1}{2} \frac{\omega \rho_m}{A} \frac{\text{Im}[-f_v]}{|1 - f_v|^2} |U_1|^2 - \frac{1}{2} \frac{\gamma - 1}{\gamma} \frac{\omega A \text{Im}[-f_k]}{p_m} |\tilde{p}_1|^2 + \frac{1}{2} \times \frac{1}{T_m} \frac{dT_m}{dx} \text{Re} \left[\frac{f_k - f_v}{(1 - f_v)(1 - \sigma)} \tilde{p}_1 U_1 \right], \quad (8)$$

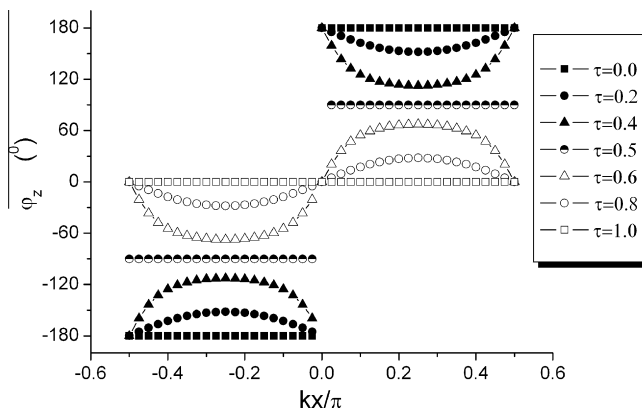


Fig. 5. φ_z versus kx/π with different τ : 0.0, 0.2, 0.4, 0.5, 0.6, 0.8, 1.0.

where σ is the Prandtl number, γ is the ratio of isobaric specific heat to isochoric specific heat, p_m is the mean pressure of the fluid, U_1 is the volume flow rate, and f_v, f_k are the cross-sectional averaged Rott's functions which depend on the specific channel geometry under consideration.

The functions of f_v and f_k are known for many geometries. As shown in Fig. 5.14 of Ref. [11], the solutions for most of these geometries are actually very similar. Therefore, this paper only takes the parallel-plate type of regenerator as a case to investigate, and other types of regenerator can be analyzed in the similar way. $y = 0$ is defined at the centerline of the spacing between two adjacent parallel plates,

$$f_v = \frac{\tan h[(1 + i)y_0/\delta_v]}{(1 + i)y_0/\delta_v}, \quad (9)$$

$$f_k = \frac{\tan h[(1 + i)y_0/\delta_k]}{(1 + i)y_0/\delta_k}, \quad (10)$$

where $2y_0$ is the thickness of each spacing, and $r_h = y_0$.

The relative hydraulic radius is defined as $\zeta = r_h/\delta_k$, the ratio of the hydraulic radius r_h to the thermal penetration depth δ_k . The functions of f_v and f_k can be written as follows,

$$f_v = \frac{\tan h[(1 + i)\zeta\sqrt{1/\sigma}]}{(1 + i)\zeta\sqrt{1/\sigma}}, \quad (11)$$

$$f_k = \frac{\tan h[(1 + i)\zeta]}{(1 + i)\zeta}. \quad (12)$$

The first two terms in Eq. (8) are always negative, which consume the acoustic power, and are independent of temperature gradient along x . The first term describes the viscous dissipation of sound wave. It is proportional to $|U_1|^2$ and independent of thermal conductivity. The second term describes the thermal-relaxation dissipation. It is proportional to $|p_1|^2$ and is independent of viscosity. The third term is called as the source/sink term, because it can either produce or consume acoustic power. It exists only if $dT_m/dx \neq 0$. For thermoacoustic engine, it is positive because the thermal energy is converted to the acoustic energy. It can be written as

$$E_g = \frac{1}{2} \left(\frac{1}{T_m} \frac{dT_m}{dx} \right) |p_1| |U_1| |e_g| \cos(\varphi_e - \varphi_z), \quad (13)$$

where

$$e_g = \frac{f_k - f_v}{(1 - f_v)(1 - \sigma)} = |e_g| e^{i\varphi_e}, \quad (14)$$

and φ_e is the phase of e_g .

As shown in Eqs. (8) and (13), the maximum power gain $d\dot{E}_2/dx$ of the TAE appears at $\varphi_z = \varphi_e$ when $dT_m/dx > 0$. Fig. 6 shows φ_e versus ζ , and $-64^\circ \leq \varphi_e < 0^\circ$. So, the optimal leading phase appears in the region of $-64^\circ \leq \varphi_z = \varphi_e(\zeta) \leq 0^\circ$, corresponding to the Case (1) of Fig. 4.

As shown in Fig. 6, with the increase of ζ , $|e_g|$ decreases. For the idea regenerator without viscous dissipation, when $\zeta \rightarrow 0$, $\varphi_e \rightarrow 0^\circ$ and $|e_g|$ reaches the maximum $|e_g| \rightarrow 1.0$. So, $d\dot{E}_2/dx$ reaches the maximum at the traveling wave phase $\varphi_z = \varphi_e = 0^\circ$. However, for the practical regenerator, the viscous dissipation cannot be neglected, and sometime is the main dissipation. So the moderately small hydraulic radius is adopted in the practical traveling wave device. It leads φ_e to deviate from 90° by thermo-relaxation, which indicates that the optimal leading phase deviates from the traveling wave phase, $\varphi_z = \varphi_e \neq 0^\circ$. So the optimal φ_z is not the traveling wave phase but traveling-standing wave phase with a negative value $-64^\circ \leq \varphi_z = \varphi_e(\zeta) \leq 0^\circ$.

In order to yield confidence in the analysis, we now give some numerical results about acoustic power amplification comparing

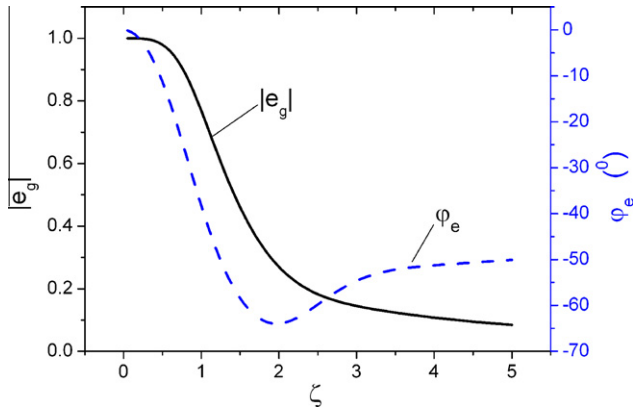


Fig. 6. $|e_g|$ and φ_e versus ζ .

with the experimental data of Biwa et al. [4] who measured the acoustic power amplification under different φ_z .

Acoustic power amplification can be expressed as,

$$\frac{E_{out}}{E_{in}} = \frac{E_{in} + \Delta E}{E_{in}} = 1 + \frac{\Delta E}{0.5|p_1||u_1|\cos(\varphi_z)}, \quad (15)$$

Substituting Eq. (8) into Eq. (15)

$$\frac{E_{out}}{E_{in}} = 1 + \frac{2\pi l}{\lambda \cos(\varphi_z)} \left\{ -\frac{\text{Im}[-f_v]}{|1-f_v|^2} \times \frac{1}{|Z_N|} - (\gamma - 1)\text{Im}[-f_k] \times |Z_N| \right\} + \frac{\epsilon}{\omega} \frac{1}{T_m} \frac{dT_m}{dx} |e_g| \cos(\varphi_e - \varphi_z) \quad (16)$$

According to the experiment of Ref. [4] and Eq. (16), the acoustic power amplifications versus φ_z are calculated when $\zeta = 0.36$ and 1.87 , as shown in Fig. 7. The theoretical curves have the similar shapes as corresponding experimental curves, showing a general agreement between the theoretical and experimental results. Thus, we can explain the experimental results with the viewpoint discussed above.

As shown in Fig. 7a, when $\Delta T_m > 0$, E_{out}/E_{in} shows a maximum reaching 1.7 at $\varphi_z \approx -42^\circ$. This corresponds to the situation where the hot end of the regenerator is located close to the velocity node. Biwa et al. [4] explained this phenomenon as follows: “Since the kinematic viscosity μ of the gas increases with increasing temperature, viscous losses should become relatively larger at the hot end than that at the cold end, even if the velocity amplitude is the same at both ends of the regenerator. Thus, $\Phi_m = -\varphi_z$ yielding minimum viscous losses should be positive so that the velocity node comes close to the hot end. Therefore, we consider that the gain shows a maximum at $\Phi_m = -\varphi_z > 0^\circ$ instead of $\Phi_m = -\varphi\varphi_z = 0^\circ$.” However, calculations in this paper use the average values of regenerator parameters, and neglect the changing of μ with the temperature. The maximum acoustic power amplification also appears at $\Phi_m = -\varphi_z > 0^\circ$. That reason is that φ_e deviates from 0° to a certain value between 0° and -90° because of $\zeta = 0.36$. So the optimal value of $-90^\circ < \varphi_z < 0^\circ$ is chosen to match the deviation of φ_e . When $\Delta T_m > 0$ and $\varphi_z < 0^\circ$, this corresponds to the arrangement as Case (1) in Fig. 4, which is the best arrangement for the TAE. The regenerator realizes the conversion from heat to acoustic energy by both the SWC and the TWC. As shown in Fig. 7a, when $\Delta T_m < 0$, the maximum of E_{out}/E_{in} appears in the region of $\varphi_z > 0^\circ$. For this phenomenon, Biwa et al. [4] gave the similar explanation for the phenomenon when $\Delta T_m < 0$: “ $\Phi_m = -\varphi_z$ yielding minimum viscous losses should be negative so that the velocity node comes close to the hot end.” However, calculations in this paper use the average values of the regenerator’s parameters, and neglect the changing of μ with the temperature. The maximum acoustic power

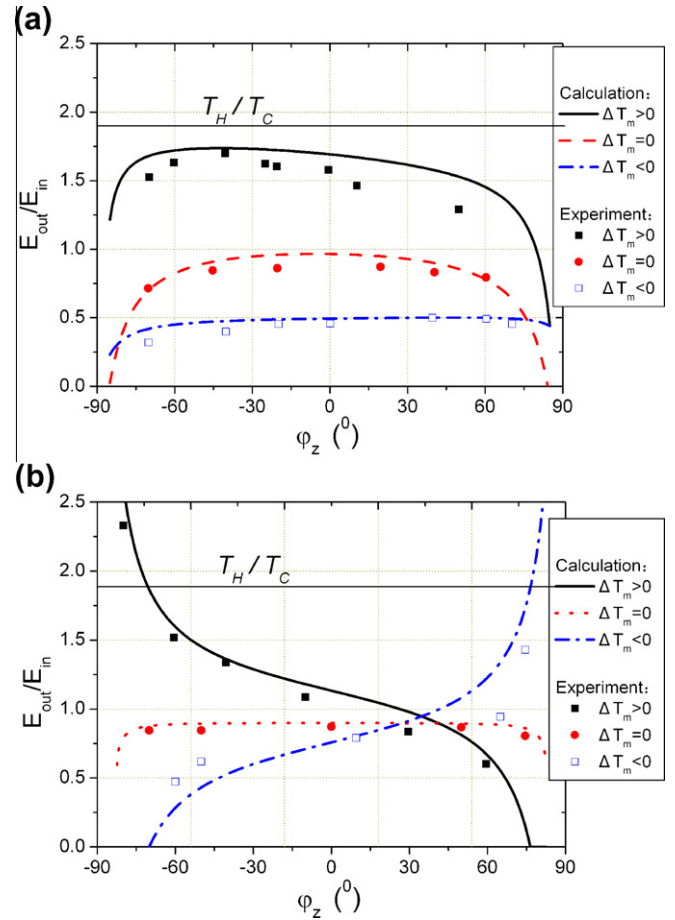


Fig. 7. E_{out}/E_{in} versus φ_z for a variety of ΔT_m and ζ : (a) $\zeta = 0.36$, (b) $\zeta = 1.87$.

amplification also appears at $\Phi_m = -\varphi_z < 0^\circ$. That reason is that $\varphi\varphi_e$ deviates from 0° to a certain value between 0° and -90° because of $\zeta = 0.36$. Thus the value of $\cos(\varphi_e - \varphi\varphi_z) = \cos \varphi_e \cos \varphi_z + \sin \varphi_e \sin \varphi_z$ in the region of $90^\circ > \varphi_z > 0^\circ$ is smaller than that in the region of $0^\circ > \varphi_z > -90^\circ$. This indicates that the acoustic power is smaller in the region of $90^\circ > \varphi_z > 0^\circ$.

As shown in Fig. 7b, the curve of E_{out}/E_{in} versus φ_z when $\zeta = 1.87$ is clearly different from that when $\zeta = 0.36$. When $\Delta T_m > 0$, E_{out}/E_{in} monotonically decreases with the increase of $\varphi\varphi_z$ and it exceeds $T_H/T_C = 1.9$ when $\varphi_z < -60^\circ$. When $\zeta = 1.87$, the thermal-relaxation dissipation becomes the main loss, and the acoustic power gain is mainly dependent on the thermoacoustic conversion of the SWC. In the standing wave engine, $E_{in} \rightarrow 0.0$ and acoustic energy can be generated within the regenerator. This represents that $E_{out}/E_{in} \rightarrow \infty$ when the energy conversion is mainly dependent on the SWC. Thus, the results clearly demonstrate that amplification of E_{out}/E_{in} exceeding T_H/T_C is possible through the SWC energy conversion in the regenerator with $\zeta = 1.87$ [4].

Fig. 7 can also be explained through the analysis of Section 2 in this paper. When $\Delta T_m > 0$ and $\varphi\varphi_z < 0^\circ$, the regenerator arrangement is the same as Case (1) of Fig. 4: The regenerator realizes the conversion from heat to acoustic energy by both the SWC and the TWC. Thus, $E_{out}/E_{in} > 1.0$ in the region of $\varphi_z < 0^\circ$, as shown in Fig. 7. When $\Delta T_m > 0$ and $\varphi_z > 0^\circ$, the regenerator arrangement is the same as Case (2) of Fig. 4: The SWC consumes acoustic energy, while the TWC converts heat to acoustic energy. When the thermoacoustic effect induced by the TWC is stronger than that by the SWC, the regenerator can produce acoustic energy. Thus, $E_{out}/E_{in} > 1.0$ appears in the region of $\varphi_z > 0^\circ$, as shown in Fig. 7. When

the thermoacoustic effect induced by the SWC is stronger than that by the TWC, the regenerator contributes to the damping of acoustic energy. Thus, $E_{out}/E_{in} < 1.0$ also appears in the region of $\varphi_z > 0^\circ$, as shown in Fig. 7. When $\Delta T_m < 0$ and $\varphi_z > 0^\circ$, the regenerator arrangement is the same as Case (3) of Fig. 4: the SWC converts heat to acoustic energy, while the TWC consumes acoustic energy. When the thermoacoustic effect induced by the SWC is stronger than that by the TWC, the regenerator can produce acoustic energy. Thus, $E_{out}/E_{in} > 1.0$ appears in the region of $\varphi_z > 0^\circ$, as shown in Fig. 7. When the thermoacoustic effect induced by the TWC is stronger than that by the SWC, the regenerator contributes to the damping of acoustic energy. Thus, $E_{out}/E_{in} < 1.0$ also appears in the region of $\varphi_z > 0^\circ$, as shown in Fig. 7. When $\Delta T_m < 0$ and $\varphi_z < 0^\circ$, the regenerator arrangement is the same as Case (4) of Fig. 4: both the SWC and the TWC consume acoustic energy. Thus, $E_{out}/E_{in} < 1.0$ in the region of $\varphi_z < 0^\circ$, as shown in Fig. 7.

According to the analysis above, the optimum value of φ_z is not only dependent on the changing of μ with the temperature, but also mainly dependent on the phase deviation because of the thermo-relaxation with the finite hydraulic radius. The optimum value of φ_z must match the phase deviation due to the thermo-relaxation. It is found that the optimum traveling-standing wave phase φ_z can also make both the TWC and the SWC contribute to the thermoacoustic conversion.

4.2. Heat flow

In the regenerator model, ordinary thermal conduction in x direction is neglectable. The average heat flow generated along regenerator due to hydrodynamics transport is [1,12]

$$\dot{Q} = \frac{|U_1|^2 \rho_m c_p}{2A\omega(1-\sigma^2)|1-\tilde{f}_v|^2} \frac{dT_m}{dx} \text{Im}(f_k + \sigma\tilde{f}_v) - \frac{1}{2} \text{Re} \left[p_1 \tilde{U}_1 \left(\frac{f_k - \tilde{f}_v}{(1+\sigma)(1-\tilde{f}_v)} \right) \right], \quad (17)$$

When $dT_m/dx > 0$, the first term in Eq. (17) is negative, which transports undesired heat from hot end to cold end, and is independent of φ_z . The first term describes the acoustically induced heat convection. The second term in Eq. (17) can either pump heat from cold to hot or transport heat from hot to cold. For the TAC, it is positive when $dT_m/dx > 0$, because the acoustic wave pumps heat from cold to hot. It can be written as,

$$\dot{Q}_g = -\left(\frac{1}{2}|p_1||U_1|\right)|q_g| \cos(\varphi_q + \varphi_z), \quad (18)$$

where

$$q_g = \frac{f_k - \tilde{f}_v}{(1+\sigma)(1-\tilde{f}_v)} = |q_g| e^{i\varphi_q}, \quad (19)$$

and φ_q is the phase of q_g .

As shown in Eq. (18), the maximum cooling power $\dot{Q}$ of the TAC appears at $\varphi_z = 180^\circ - \varphi_q$ when $dT_m/dx > 0$. Fig. 8 shows φ_q versus ζ , and $-84^\circ < \varphi_q < 0$. So, the optimal leading phase appears in the region of $-180^\circ < \varphi_z = 180^\circ - \varphi_q < -96^\circ$, corresponding to the Case (3) of Fig. 4.

4.3. Second law efficiency of the TAE

The second law efficiency η_2 is the first law efficiency η_1 divided by the Carnot efficiency η_c ,

$$\eta_2 = \eta_1/\eta_c. \quad (20)$$

According to Eqs. (8), (17), and (20), it can be yielded,

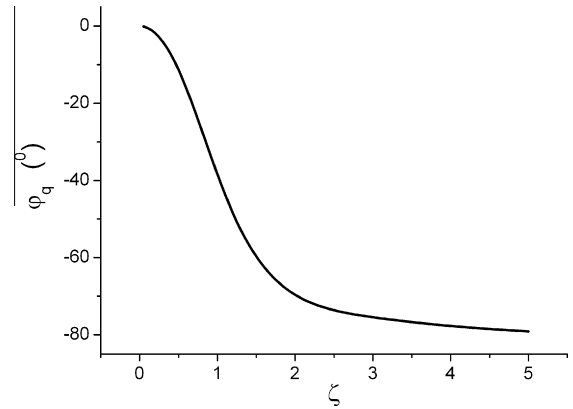


Fig. 8. φ_q versus ζ .

$$\eta_2 = \frac{\eta_1}{\eta_c} = \frac{\Delta \dot{E}_2 / \dot{Q}_h}{\Delta T_m / T_m} = \frac{\frac{d\dot{E}_2/dx}{-Q}}{\frac{1}{T_m} \frac{dT_m}{dx}} = -\frac{\frac{1}{\psi} \left(-\frac{\text{Im}[-\tilde{f}_v]}{|Z_N||1-\tilde{f}_v|^2} - |Z_N|(\gamma-1)\text{Im}[-f_k] \right) + |e_g| \cos(\varphi_e - \varphi_z)}{\frac{\psi}{|Z_N||1-\tilde{f}_v|^2(1-\sigma^2)(\gamma-1)} \text{Im}(f_k + \sigma\tilde{f}_v) - |q_g| \cos(\varphi_q + \varphi_z)}, \quad (21)$$

where ψ is the operation factor of the regenerator, defined as

$$\psi = \frac{c}{\omega} \frac{1}{T_m} \frac{dT_m}{dx}, \quad (22)$$

η_2 has been calculated for a range of φ_z and ζ according to Eq. (21) on a minicomputer having a FORTRAN compiler capable of evaluating complex equations. In the calculations, we use the values of the thermoacoustic Stirling heat engine described in the Ref. [13], such as $|Z_N| = 15$, $\psi = 22$ and helium as working gas.

Fig. 9 shows the relation between η_2 and φ_z with different ζ . With the increase of ζ , the maximal η_2 of each ζ increases at first, reaches the maximal value 0.786 at the point of ($\zeta = 0.21$, $\varphi_z = -25.2^\circ$), and then decreases. In all cases of ζ , η_2 reaches the maximum in the region of $-90^\circ < \varphi_z < 0$, which is relative to Case (1) in Fig. 4. When $\zeta \rightarrow 0$, the optimal φ_z is close to 0° . However, $\eta_2 \rightarrow 0$, because viscous losses should become infinite when

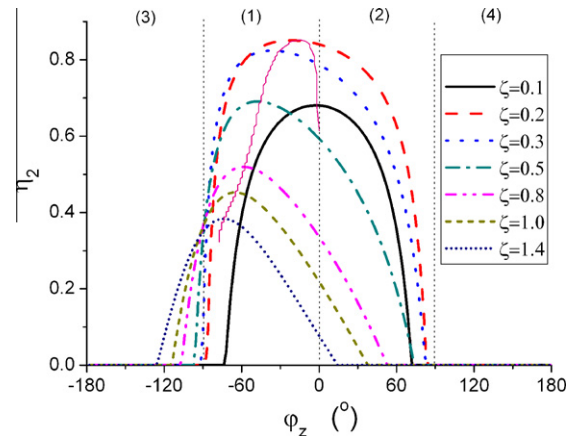


Fig. 9. η_2 versus φ_z for a variety of ζ . The fine solid line connects the optimal points of different ζ .

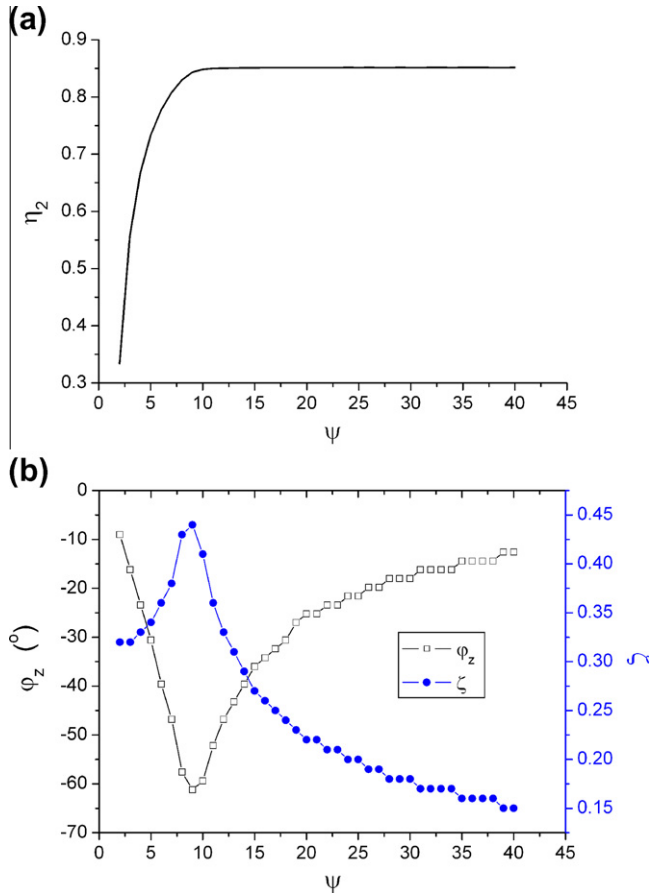


Fig. 10. (a) The maximum η_2 with different ψ . (b) The optimal ζ and the optimal φ_z for the maximum η_2 with different ψ .

$\zeta \rightarrow 0$. With the increase of ζ , the optimal φ_z deviates from 0° to a certain value between 0° and -90° .

Furthermore, the maximum η_2 , the optimal ζ and the optimal φ_z with different ψ are summarized in Fig. 10. With the increase of ψ , the maximum η_2 increases sharply at first, and then increases slowly and gradually reaches to $\eta_2 = 0.85$. As shown in Fig. 10b, as the optimal ζ deviates from 0.0, the optimal φ_z deviates from 0° because of the thermo-relaxation. Although the range of the optimal φ_z is large, all of the optimal φ_z is located in the range of -90° to 0° .

On the other hand, the maximum η_2 , the optimal ζ and the optimal φ_z with different $|Z_N|$ are summarized in Fig. 11. Similarly to the phenomenon of ψ , with the increase of $|Z_N|$, the maximum η_2 increase, and then reaches to $\eta_2 = 0.94$. The optimal phase difference φ_z depends on the optimal ζ because the phase difference needs to balance the phase deviation generated by thermo-relaxation. Similarly, all of the optimal φ_z is located in the range of -90° to 0° .

As a result, the TAE's regenerator arranged as Case (1) is the best.

4.4. Second law coefficient-of-performance of the TAC

The second law coefficient of performance COP_2 is the first law coefficient of performance COP_1 divided by the Carnot coefficient of performance COP_c

$$COP_2 = COP_1 / COP_c. \quad (23)$$

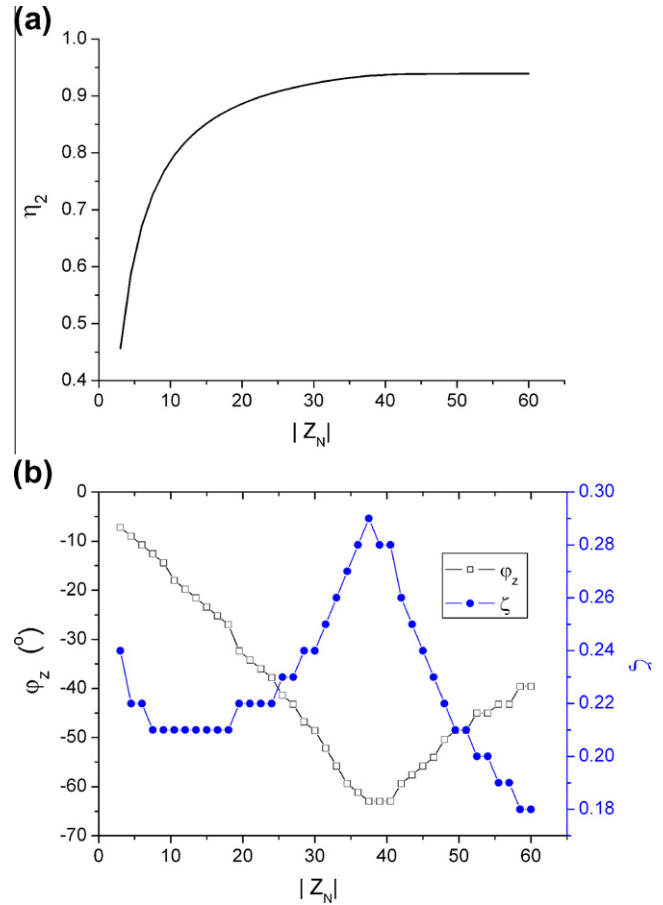


Fig. 11. (a) The maximum η_2 with different $|Z_N|$. (b) The optimal ζ and the optimal φ_z for the maximum η_2 with different $|Z_N|$.

According to Eqs (8), (17), and (23),

$$\begin{aligned} COP_2 &= \frac{\frac{\dot{Q}_c}{-T_m}}{\frac{\dot{Q}}{\Delta T_m}} = \frac{\frac{\dot{Q}}{-dE_2/dx}}{\frac{\dot{Q}}{dT_m/dx}} \\ &= -\frac{\frac{\psi}{|Z_N||1-f_v|^2(1-\sigma^2)(\gamma-1)} \text{Im}(f_k + \sigma \tilde{f}_v) - |q_g| \cos(\varphi_q + \varphi_z)}{\frac{1}{\psi} \left(-\frac{\text{Im}[-f_v]}{|Z_N||1-f_v|^2} - |Z_N|(\gamma-1) \text{Im}[-f_k] \right) + |e_g| \cos(\varphi_e - \varphi_z)}. \end{aligned} \quad (24)$$

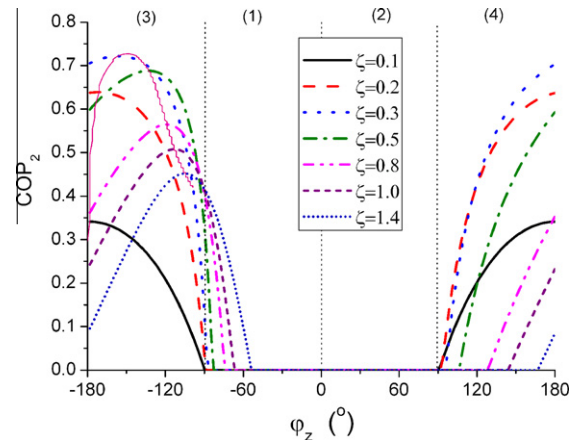


Fig. 12. COP_2 versus φ_z for a variety of ζ . The fine solid line connects the optimal points of different ζ .

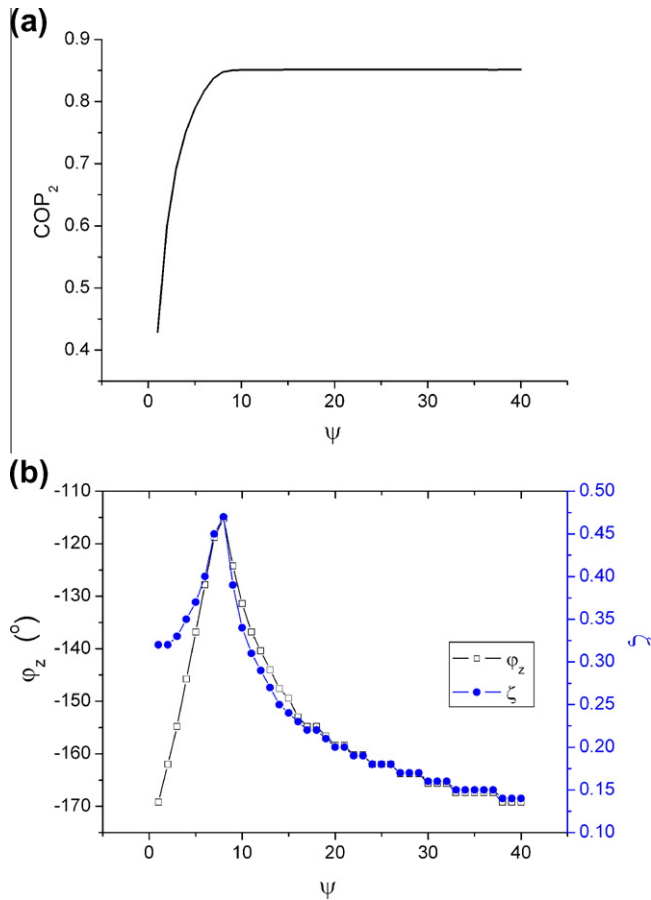


Fig. 13. (a) The maximum COP₂ with different ψ . (b) The optimal ζ and the optimal ϕ_z for the maximum COP₂ with different ψ .

COP₂ has been calculated for a range of ϕ_z and ζ values according to Eq. (24) on a minicomputer having a FORTRAN compiler capable of evaluating complex equations. In the calculations, we let $|Z_N| = 15$, $\psi = 3.6$ and use helium as working gas according to the thermoacoustic Stirling cooler described in Ref. [14].

Fig. 12 shows the relation between COP₂ and ϕ_z with different ζ . With increase of ζ , the maximal COP₂ of each ζ increases at first, reaches the maximal value 0.85 at the point of ($\zeta = 0.24$, $\phi_z = -149.4^\circ$), and then decreases. In all cases of ζ , COP₂ reaches maximum in the region of $-180^\circ < \phi_z < -90^\circ$, which is relative to the Case (3) in Fig. 4. When $\zeta \rightarrow 0$, the optimal ϕ_z is close to -180° . However, COP₂ $\rightarrow 0$, because viscous losses should become infinite when $\zeta \rightarrow 0$. With the increase of ζ , the optimal ϕ_z deviates from -180° to a certain value between -180° and -90° .

Furthermore, the maximum COP₂, the optimal ζ and the optimal ϕ_z with different ψ are summarized in Fig. 13. With the increase of ψ , the maximum COP₂ increase sharply at first, and then increase slowly and gradually reaches to COP₂ = 0.85. As shown in Fig. 13b, as the optimal ζ increases from 0.0, the optimal ϕ_z increases from -180° because of the thermo-relaxation. Although the range of the optimal ϕ_z is large, all of the optimal ϕ_z is located in the range of $-180^\circ \sim -90^\circ$.

On the other hand, the maximum COP₂, the optimal ζ and the optimal ϕ_z with different $|Z_N|$ are summarized in Fig. 14. Similarly to the phenomenon of ψ , with the increase of $|Z_N|$, the maximum COP₂ increase, and then reaches to COP₂ = 0.73. The optimal phase difference ϕ_z depends on the optimal ζ because the phase difference needs to balance the phase deviation generated by thermo-relaxation. Similarly, all of the optimal ϕ_z is located in the range of -180° to -90° .

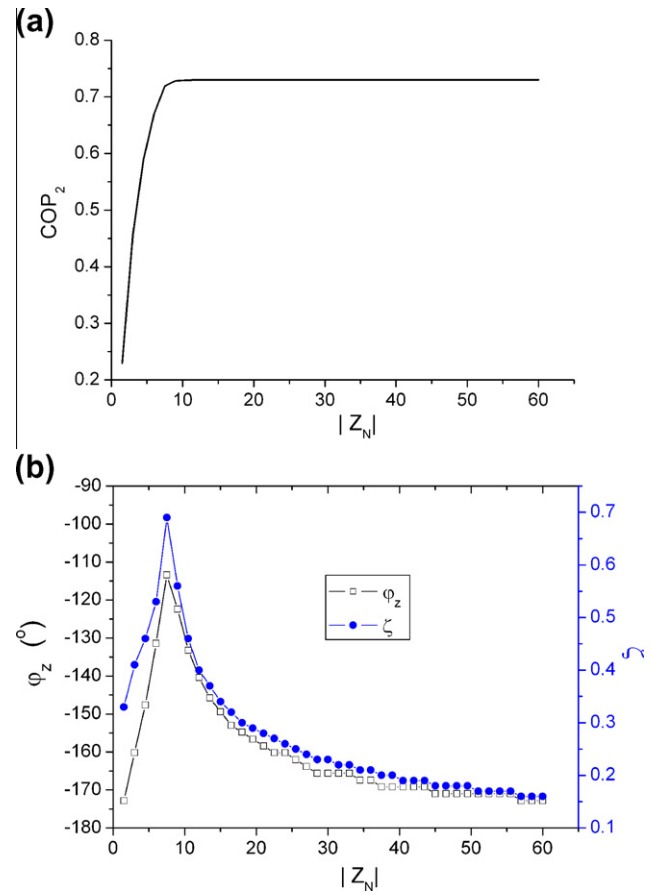


Fig. 14. (a) The maximum COP₂ with different $|Z_N|$. (b) The optimal ζ and the optimal ϕ_z for the maximum COP₂ with different $|Z_N|$.

As a result, the TAC's regenerator arranged as Case (3) is the best.

5. Conclusions

Actually, the acoustic field in practical thermoacoustic devices is neither a pure traveling wave nor a pure standing wave. The thermoacoustic effect is the hybrid effect of traveling wave component and standing wave component. Thus, the thermoacoustic effect of the traveling-standing wave is investigated in this paper. Firstly, the thermoacoustic conversion properties of the traveling-standing wave are analyzed qualitatively by combining the thermoacoustic conversion properties of the traveling wave with those of the standing wave. Then, according to the basic thermoacoustic formulas, this paper analyzes the optimum phase difference which can make both the traveling wave and the standing wave contribute to the thermoacoustic conversions. The results are consistent with the qualitative analysis: to gain a higher acoustic power and efficiency, the hot end of the regenerator in a thermoacoustic engine should be close to the pressure antinode, and the traveling wave component should propagate from the ambient end to the hot end. To gain a higher cooling power and coefficient of performance, the ambient end of the regenerator in a thermoacoustic cooler should be close to the pressure antinode, and the traveling wave component should propagate from the ambient end to the cold end.

Furthermore, this paper analyzes the influence of the parameters of the acoustic field and the regenerator's structure on the thermoacoustic conversion, and finds out the optimum condition

for the thermoacoustic conversion. The optimal phase difference between pressure and velocity depend on the optimal hydraulic radius because the phase difference needs to balance the phase deviation generated by thermo-relaxation.

Additionally, our theoretical results also show a good agreement with the experimental data [4], which validates the analysis in this paper. Besides, the analysis in this paper further provides a more intensive understanding of these experimental results. For improving thermoacoustic conversion, the optimum φ_z is not only dependent on changing of the kinematic viscosity with the temperature, but also mainly dependent on the phase deviation because of the thermo-relaxation with the finite hydraulic radius.

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